5.1.1

Steep reinforced slopes

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1 Geogrids and reinforced soil

Reinforced soil is a composite material which combines the typical resistance of two different materials in such a way to minimize the weakness of each one. Particularly, a relative large quantity of the cheapest and compression resistant material, the soil, is improved in its engineering characteristics by the combination with a relatively small quantity of a more expensive and highly tensile resistant material, the geogrids. Thus, a sinergy is developed between the tensile and compressive resistance of the two materials: this fact improves the global characteristics of the composite material, like with concrete and steel (Fig. 1).

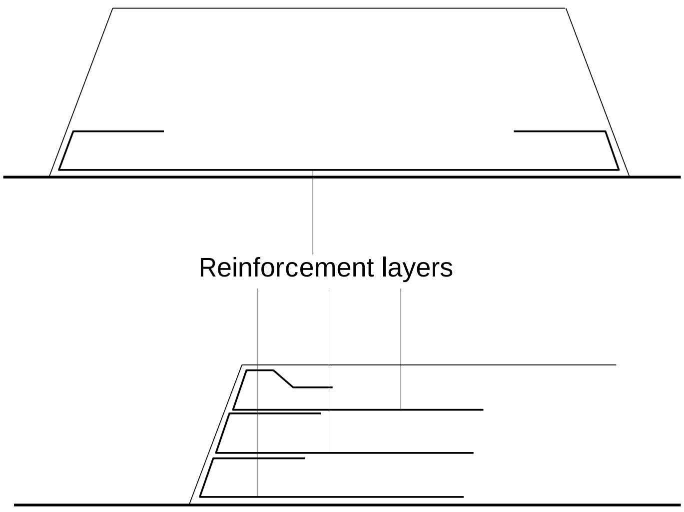


Fig. 1 - Typical base and slope reinforcement for an embankment

A relevant number of projects has already been realized worldwide, allowing the development of the techniques of design and construction of reinforced soil.

2 Basic theory of reinforced soil

A simple model helps to explain the principle on which the reinforced soil techniques are based (Jewell, 1980).
Let us consider the soil element in Fig. 2a, which is part of an infinite mass of soil: the application of a vertical stress σ_v causes a deformation in the element and the consequent horizontal stress σ_h caused by the lateral compression suffered by the adjacent soil. Horizontally the soil element undergoes a "tensile deformation" ε_h, which is one of the principal causes of local failure.
When, as in Fig. 2b, a reinforcing element is put in the soil, the application of a vertical stress is followed by the deformation of the soil element and the extension of the reinforcement.

This extension then generates a tensile strength T in the reinforcement, which in turn produces a horizontal stress σ_h*. This stress, which also provides a confinement action on the soil granules, greatly contributes to resist the horizontal forces and to reduce the horizontal deformations.

Therefore the inclusion of a geogrid into the soil mass reduces the stresses and strains applied to the soil; or, on the other hand, the vertical stress σ_v applied to the soil mass can be increased, compared to the unreinforced soil, at equal deformations.

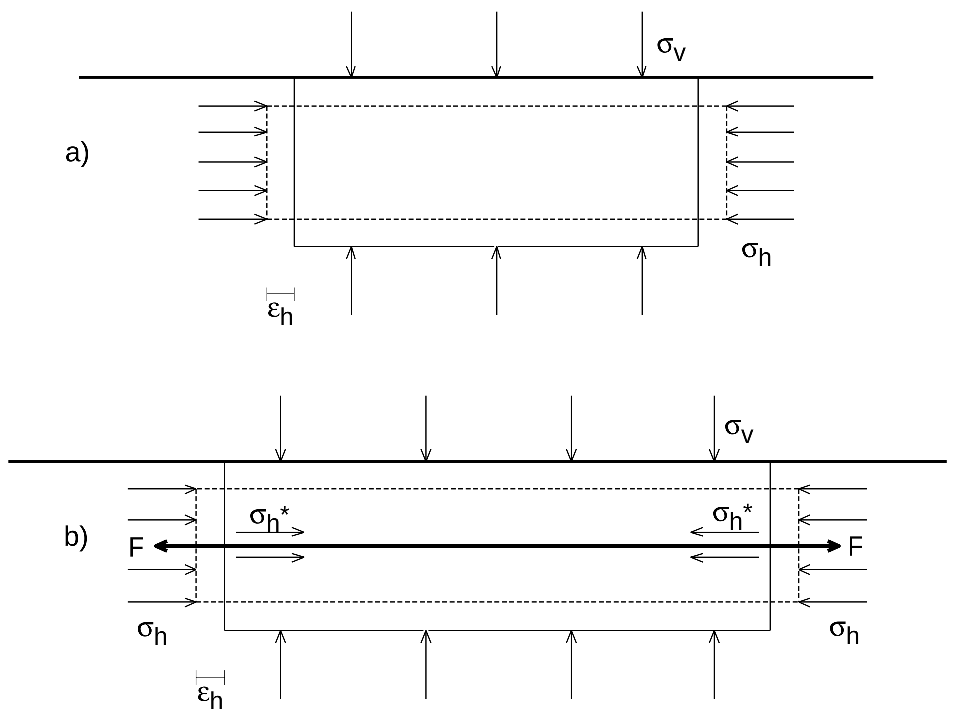


Fig. 2 - Stresses and strain in an unreinforced and a reinforced soil element

With regards to the resistance to the shear stresses, according to Fig. 3 in a non-cohesive soil element we have:

$$\left(\tau_{yx}\right)_{\max} = \sigma_y \cdot \tan\phi_{\max} \qquad (1)$$

where ϕ_{max} = maximum angle of shear resistance of soil;
$(\tau_{yx})_{max}$ = maximum overall shear stress provided by the soil.

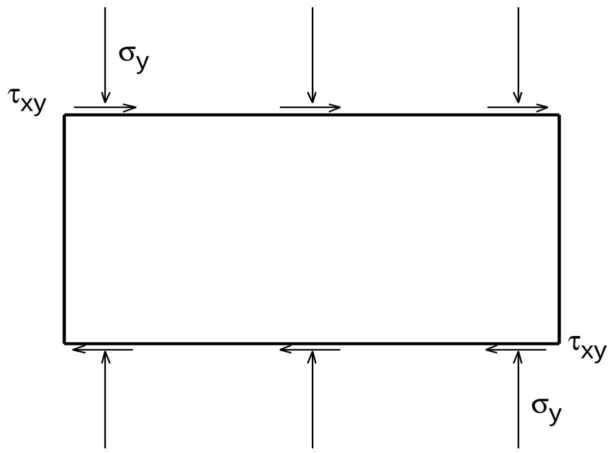


Fig. 3 - Shear stresses in an unreinforced soil element

When the soil element is crossed by a reinforcement element which makes a θ angle with the shearing direction (fig. 4), the state of stress is modified because the tension T generates a shear stress produced by the tangential component T·senθ, meanwhile the normal component T·cosθ generates another $\hat{\tau}_{yx}$ caused by the friction angle ϕ_{max} in the soil (Jewell, 1980).

Therefore:

$$\left(\tau_{yxr}\right)_{\max} = \sigma_{yr} \cdot \tan\phi_{\max} + \left(T / A_S\right) \cdot \cos\theta \cdot \tan\phi_{\max} + \left(T / A_S\right) \cdot \sin\theta \qquad (2)$$

Total shear resistance	=	shear resistance of soil alone	+	shear stress caused by the normal component of T	+	shear stress caused by the tangential component of T

where A_S = area of the soil element
$(\tau_{yxr})_{max}$ = maximum overall shear stress of the reinforced soil

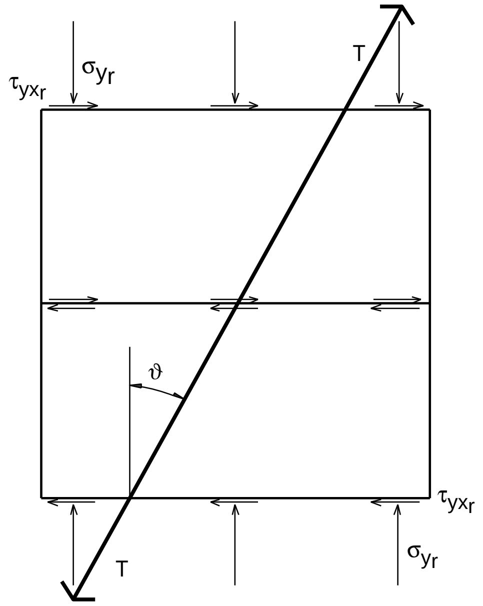


Fig. 4 - Shear stresses in a reinforced soil element

So the normal stress on the soil element is increased by:

$$\hat{\sigma}_y = (T / A_s) \cdot \cos\theta \quad (3)$$

while the maximum shear stress which the soil can carry is increased.

The main advantages of a reinforced soil structure are the following:

- lower global cost: the possibility to build with steeper slopes reduces the quantity of the material needed for an embankment;
- moreover, it is possible to use less valuable and then cheaper materials;
- improved stability: the reinforcement guarantees an improvement in the factor of safety, even in seismic areas;
- it is possible to build directly on low-bearing-capacity soils;
- a reinforcement on the base allows to build on soft soils, that would normally request a preliminary consolidation and great caution during construction.

3 Steep reinforced slopes: definition and framing of the problem

For a uniform fill soil there is a limiting slope angle β_{lim} to which an unreinforced slope may be safely built.
For the case of a non-cohesive and dry material, the limit angle of the slope equals the friction angle of the soil:

$$\beta_{\text{lim}} = \phi$$

A slope with a greater angle than the limiting slope angle is a steep slope; to build an embankment with a steep slope it is necessary to provide some addictional forces to maintain equilibrium.
The easiest method is to place horizontally some reinforcing layers in the slope so that the reinforcements can resist the horizontal forces, thus increasing the allowable shear stresses.
The forces which must be applied to the soil to maintain equilibrium can be added up in a gross force that works in a horizontal direction, that is the direction of the reinforcements.

The gross force T may be expressed with the following equation (Jewell, 1991):

$$T = 1/2 \cdot K \cdot \gamma \cdot H^2 \qquad (4)$$

where:

H = height of the slope [m]
γ = unit weight of the soil [kN/m³]
K = equivalent earth pressure coefficient, depending on the angle of the slope ß, the soil strength parameters c' and ϕ', and the pore pressure coefficient $r_u = u/(\gamma \cdot z)$.

For the case of vertical face, the coefficient K equals the coefficient of active earth pressure K_a; when β is between ϕ and 90°, K has a value between 0 and K_a.

4 Required and available forces

The additional forces required to provide equilibrium for a steep slope, with an adequate margin of safety in respect of any potential failure mechanism, can be determined by a limit equilibrium analysis. It consists in considering the possible failure surfaces in the soil and in comparing, for each of them, the active shear stresses and the resistant shear stresses in the soil.

The Factor of Safety is calculated as the ratio between the maximum resistant shear force provided by the soil an instant before failure (that is in conditions of limit equilibrium) and the active force actually developed on the considered surface.

An extensive research allows to find out the surface which yields the minimum Factor of Safety, which must be compared with the one required for design needs.

Experimental surveys (Binnie & Partners, 1982) allowed to control that the method for limit equilibrium analysis usually called ‘two-parts wedge method’ gives the best results in terms of precision versus required calculation time. This method allows to determine the forces required for equilibrium, taking into account the geometry of the slope, the geotechnical properties of the soil, the pore water pressure u, and the surcharge loading.

The reinforcement layers are calculated in such a way as to provide the required forces; it is then possible to define the envelope of the maximum available force in each layer, which depends on the bond forces between the reinforcement and the soil, on the reinforcement properties, on the tensile deformations compatible with the functionality of the work.

The diagram of the available forces must cover the one of the required forces, as shown in fig. 5.

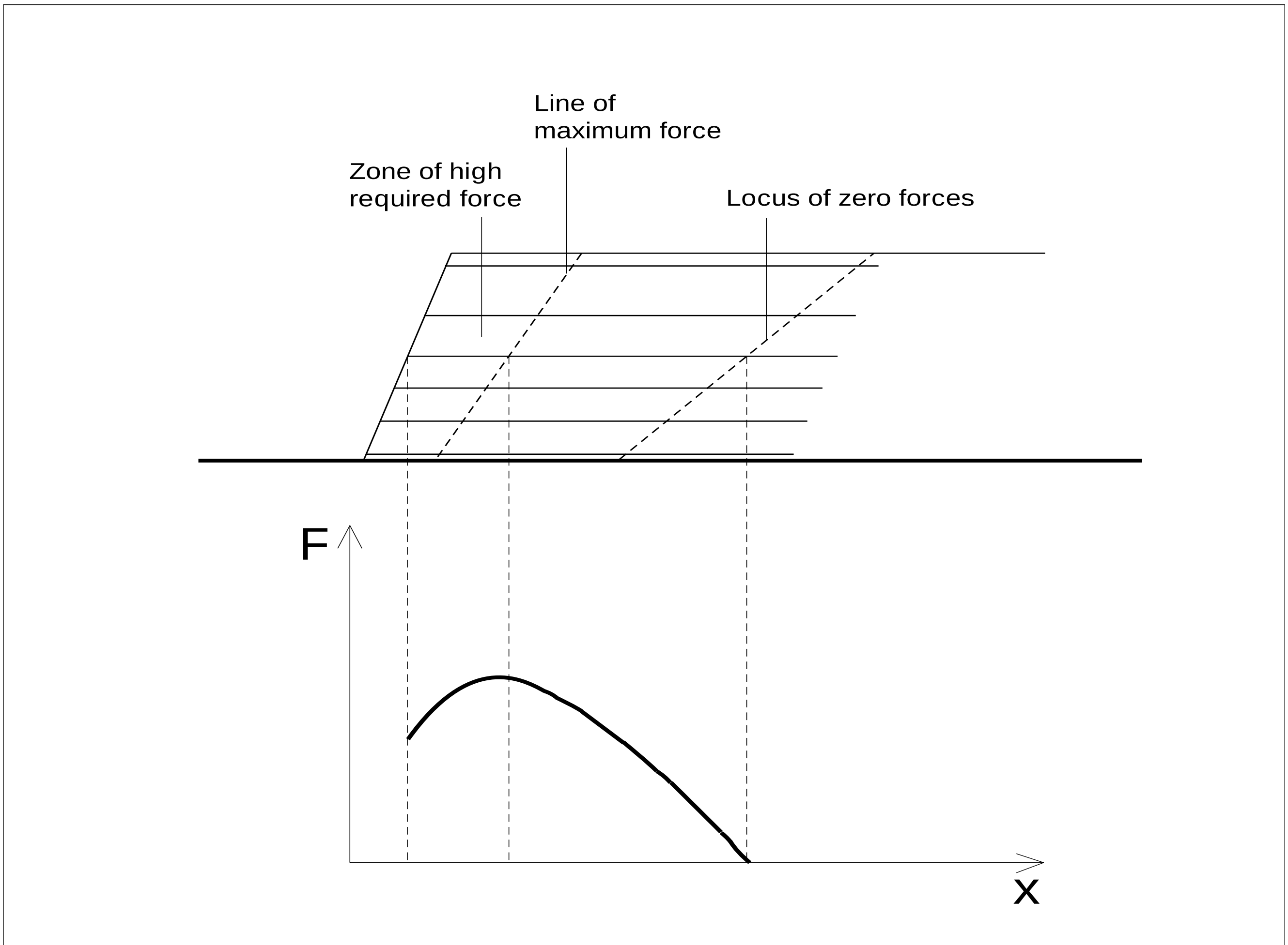


Fig. 5 - Horizontal distribution of required force for equilibrium

Once the type of reinforcement is defined, in our case the geogrids, the aim of the design is then to provide sufficient reinforcing layers, distributed in such a way that in every point of each layer the available force is higher than the required force, with the pre-defined Safety Factor.

5 Distribution of the maximum required forces

The gross horizontal force required to maintain equilibrium in every particular case of potential slip mechanism can be calculated by applying the same external forces to the slope and imposing that the slope is in equilibrium conditions, assuming a set value of the shear stress mobilised by the soil.

The analysis of the whole range of potential slip mechanisms allows to set two particular surfaces within the slope that are defined as the "line of zero required force" and the "line of maximum required force". The line of zero required force defines the zone of the soil where the reinforcement layers are required to maintain equilibrium. The line of maximum required force is the line which connects the points where the required force is the highest; it usually passes through the toe of the slope. These two lines locate a front zone in which high values of external forces are required and a zone of decreasing required force, as shown in fig. 5.

For every particular slip surface, the gross force required for the equilibrium depends on the square of the depth below the crest at which the failure surfaces emerge along the slope; we have in fact:

$$T = 1/2 \cdot K \cdot \gamma \cdot (z_i)^2 \qquad (5)$$

If we consider two very close slip surfaces (see Fig. 6), emerging at a depth dz one from the other and we assume as constant the horizontal strain σ_r required on dz, we have:

$$\sigma_r = \frac{dT}{dz} \qquad (6)$$

But σ_r must equilibrate the horizontal thrust of the soil σ_h, hence it must be:

$$\sigma_r = \sigma_h = K \cdot \gamma \cdot z \qquad (7)$$

The reinforcing layers are usually designed to support each the same force P; therefore we have that:

$$dT = \text{constant} = P \qquad (8)$$

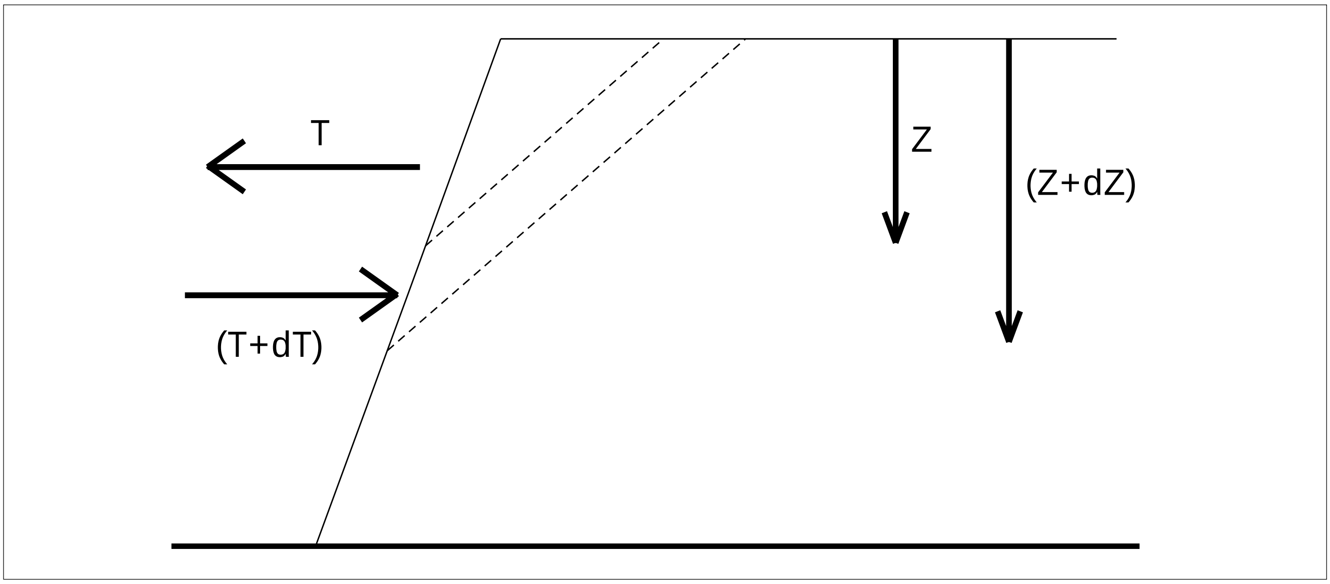


Fig. 6 - Vertical variation of the required force

To obtain this uniform force in reinforcements we have then to set the vertical spacing Sv of the reinforcing layers, in such a way as to satisfy eq. (6); therefore introducing eq. (8) the eq. becomes:

$$\sigma_r = \frac{P}{S_v} \tag{9}$$

Taking into account eq. (7), we have:

$$S_v = \frac{P}{K \cdot \gamma \cdot z} \tag{10}$$

Therefore the vertical spacing must be reduced as the depth z increases below the crest of the slope.

6 Available and allowable forces

The value of the available force at a point along a reinforcement layer depends both on the properties of the reinforcement and on the bond stresses there mobilised. The bond stresses are important to prevent two possible mechanisms of failure (Jewell et Al., 1984): the direct sliding along a reinforcement and the pull-out of the reinforcement, both caused by the thrust of the soil behind the reinforced block, as shown in fig. 7 and 8.
In the case of fig. 7 the resistance to the direct sliding along a reinforcement is a combination of shear stresses at the interface between the soil and the solid area of the geogrids and of the shear stresses at the soil-soil contacts through the apertures of the grid. The resistant shear stress is given by:

$$\tau_{ds} = \sigma_n' \cdot f_{ds} \cdot \tan\phi' \qquad (11)$$

where:

τ_{ds} = shear stress resistant to direct sliding
σ_n' = effective normal stress on the reinforcement
f_{ds} = factor of direct sliding
ϕ' = soil to soil friction angle (from direct shear tests)

The resistant force of the reinforcement is then given by:

$$T_{ds} = L \cdot B \cdot \tau_{ds} \qquad (12)$$

Typical values of f_{ds} for different types of soil and geogrids are between 0.70 ÷ 1.0 and are reported in Tab. 1.

GRAVEL	0.95 ÷ 1.00
SAND	0.92 ÷ 0.98
SILT	0.80 ÷ 0.90
CLAY	0.70 ÷ 0.80

Tab. 1 - Typical soil-geogrid direct shear coefficient f_{ds}

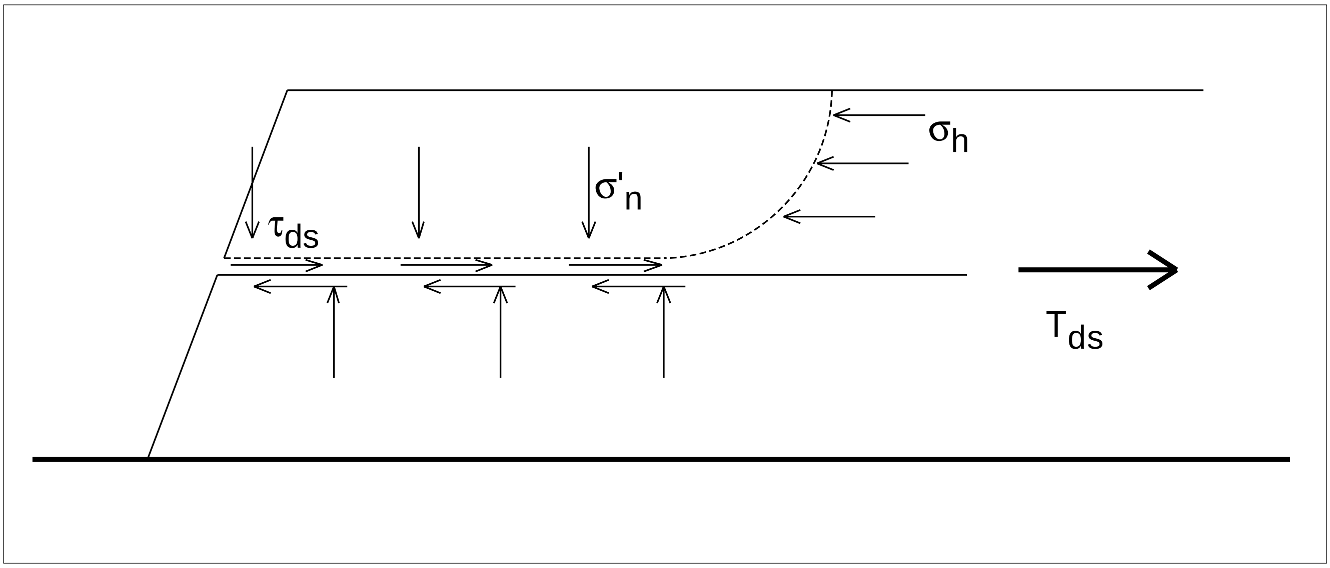


Fig. 7 - Direct sliding over a reinforcement layer

In the case of fig. 8 the resistant shear stress to the pull-out of the geogrid is:

$$\tau_b = \sigma_n' \cdot f_{po} \cdot \tan\phi' \qquad (13)$$

where: f_{po} = factor of pull-out.

The maximum resistant force in the reinforcement will be:

$$T_b = 2 \cdot L \cdot B \cdot \tau_b \qquad (14)$$

where: L, B = length and width of the reinforcement in the anchorage zone.

Typical values of f_{po} ranges between 0.8 and 1.05, and are reported in Tab. 2.

GRAVEL	0.90 ÷ 1.05
SAND	0.75 ÷ 0.95
SILT	0.70 ÷ 0.90
CLAY	0.60 ÷ 0.85

Tab 2 - Typical soil geogrid pull-out coefficient f_{po}

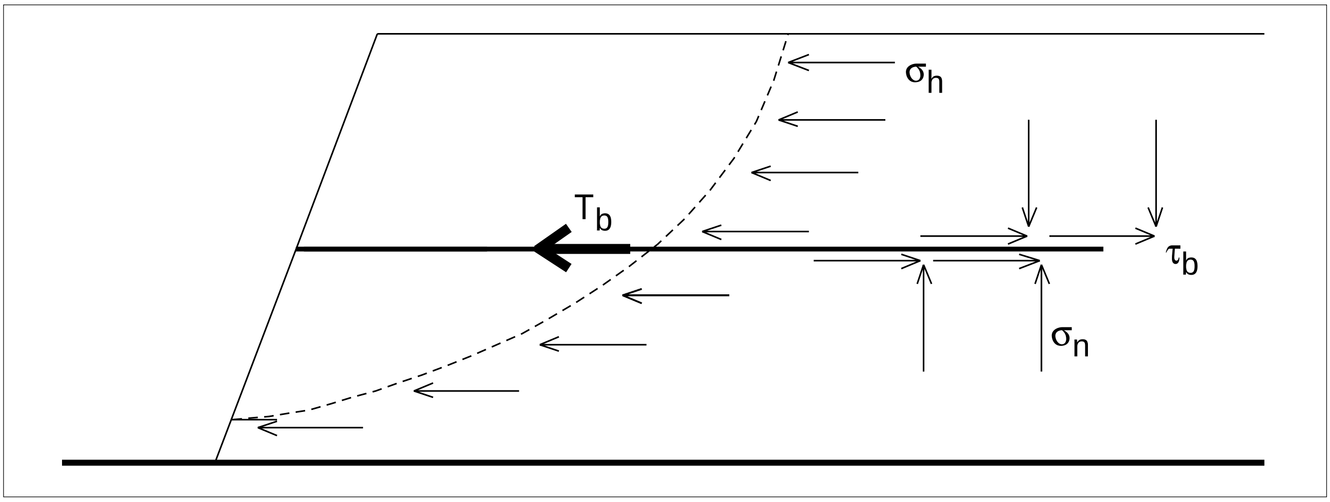


Fig. 8 - Reinforcement pull-out

If each geogrid must support the same design force P, the length of the anchorage is, for the two cases:

$$L_{ds} = \frac{P}{B \cdot \tau_{ds}} \qquad (15)$$

$$L_b = \frac{P}{2 \cdot B \cdot \tau_{ds}} \qquad (16)$$

These lengths are usually shorter than 1 metre for geogrids.

The maximum allowable force P is generally ruled by the resistance of the reinforcement or by the force in the geogrid corresponding to the maximum deformations compatible with functionality. The allowable Resistance of a geogrid is determined as a fraction of the Ultimate Strength T_{ult} by means of a Safety Factor FS_{grid}:

$$T_{allow} = \frac{T_{ult}}{FS_{grid}} \qquad (17)$$

FS_{grid} shall be obtained by multiplying several partial Factors of Safety (Koerner, 1994)

$$FS_{grid} = FS_{creep} \cdot FS_{construction} \cdot FS_{chemical} \cdot FS_{biological} \cdot FS_{junction} \qquad (18)$$

The suggested partial Factors of Safety for Tenax geogrids are summarized in Tab. 3 and 4.

TYPE OF SOIL	**PARTICLE SIZE**	**$FS_{construction}$**
silt and clay	< 0.06 mm	1.00
pulverized fuels ashes		1.00
fine and medium sand	0.06 ÷ 0.6 mm	1.00
coarse sand and fine gravel	0.6 ÷ 6 mm	1.00
crushed gravel	6 ÷ 60 mm	1.10
ballast, sharp stones		1.10

Tab 3 - $FS_{construction}$ for different types of soil

$FS_{junction}$	1.00
$FS_{chemical}$	1.00
$FS_{biological}$	1.00
FS_{creep}	2.6 ÷ 2.8

Tab. 4 - Suggested Factors of Safety for TENAX geogrids.

$FS_{junction}$ is equal to 1.00 because Tenax geogrids properties are evaluated in through the junction tests.
The biological and chemical Safety Factors are equal to 1.00 because of characteristic of HDPE, that is a polymer inert and resistant to chemical and biological attack.
The construction Safety Factor ($FS_{construction}$) is a function of the imperfections during placing, of the damage during transportation, stocking, construction and of the type of fill soil (Wright and Greenwood, 1993);
The ratio of T_{ult} by means of FS_{creep} is the Long Term Design Strength of the geogrid.

LTDS is a function of the creep phenomena in the geogrids which have an increasing importance in relation to the design life of the project: it is determined after creep tests (Montanelli & Rimoldi, 1993). Tab. 5 provides the values of LTDS for the Tenax geogrids at different temperatures.

	10°	20°	30°	40°
TT 201	19.1	16.4	13.2	11.9
TT 301	27.6	23.5	18.9	17.0
TT 401	34.0	30.6	24.5	22.0
TT 601	42.0	38.1	30.6	27.5
TT 701	46.7	42.0	33.7	30.2

Tab. 5 - LTDS in kN/m for Tenax geogrids at different temperatures

The design resistance P is determined as a fraction of the Allowable Resistance T_{allow} by means of a Safety Factor for Design (FS_{design}), which value ranges between 1.05 and 1.5.

7 Design criteria

The design problem for a reinforced slope can be set as following: once the geometry of the slope is defined, the surcharge load fixed, the geotechnical characteristics of the soil known and the design resistance P of the grids set, we must find the number, the vertical position and the length of the reinforcing layers required to provide the equilibrium for every possible failure mechanism.
The criteria to determine the unknown quantities of the problem (number, position, length of the layers) are these:

1) Each reinforcement layer must provide a sufficient force to support the horizontal stresses in the zone of soil of its competence, caused by the thrusts of the unreinforced soil behind. Referring to fig. 9, the vertical spacing S_v will have to satisfy the condition:

$$P > S_v \cdot \sigma_h \tag{19}$$

with:

$$\sigma_h = K \cdot \sigma_v \tag{20}$$

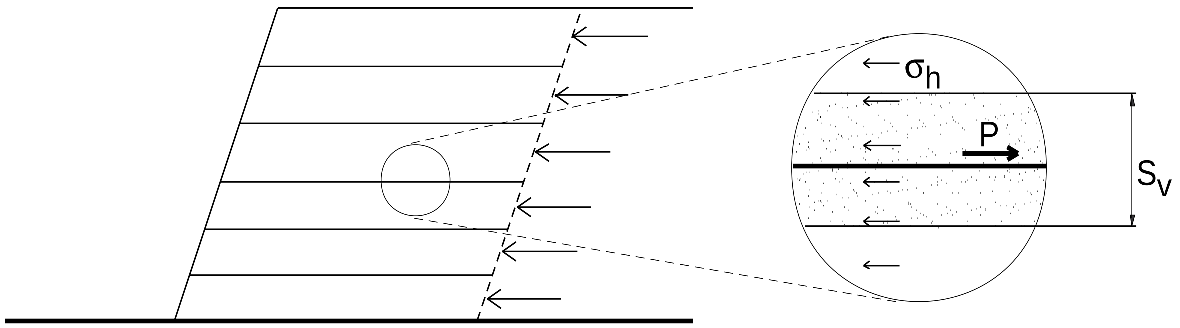
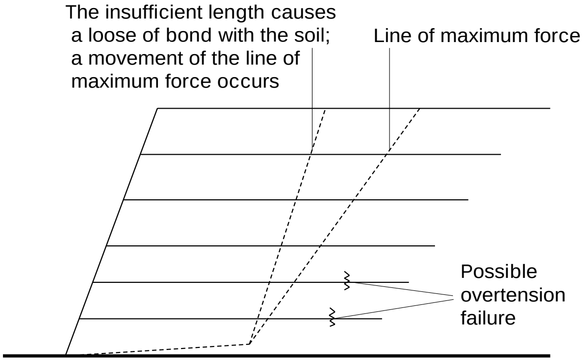


Fig.9 - Local equilibrium of a single reinforcement layer

2) Near the slope crest the geogrid length must support the whole design strength P; as shown in fig. 10 infact, if the upper layers have an insufficient length, the lower layers must resist to greater loads, that could become excessive and dangerous. This means that the anchorage length at the top shall be sufficient to avoid the pull-out of the geogrid when subjected to a tensile force equal to P.



Tab. 10 - Insufficient reinforcement length near the slope crest

3) Insufficient length near the slope base would lead to gross outward sliding of the reinforced zone along the interface between the soil and a reinforcement layer (see fig.11). Therefore the length of the geogrid at the base shall be sufficient to avoid direct sliding along any geogrid layer.

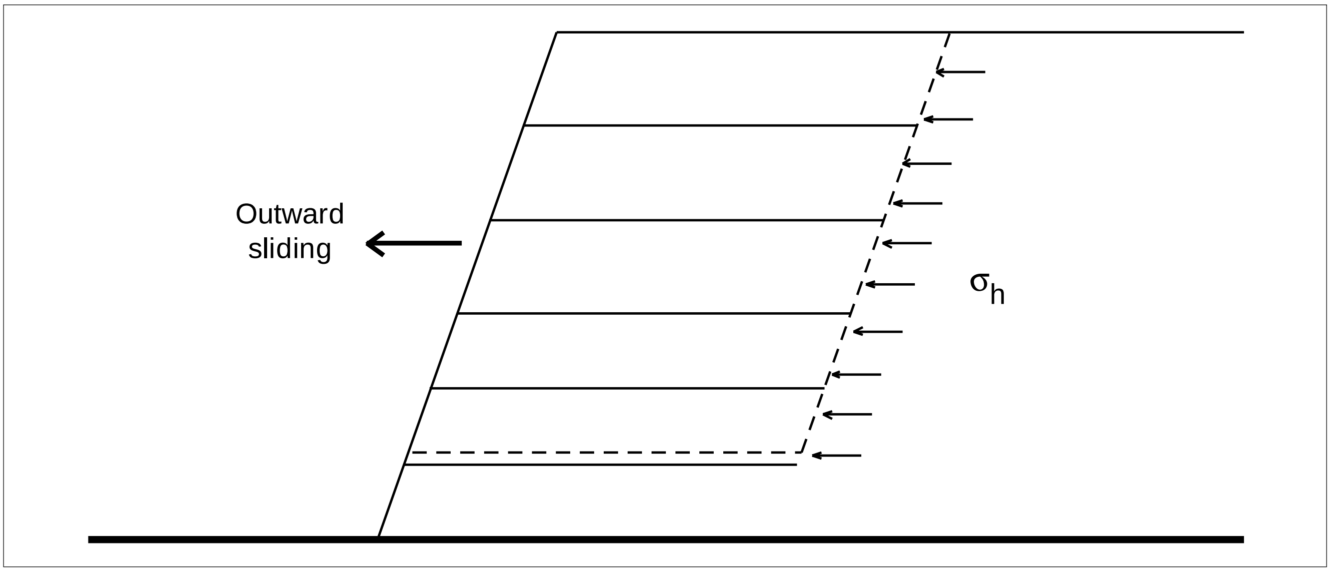


Fig. 11 - Sliding at the soil-geogrid interface

4) The reinforcement zone, acting as a rigid block should be sufficiently wide to resist the outward thrust, without developing any tensile vertical effective stress along the base, as shown in fig. 12.

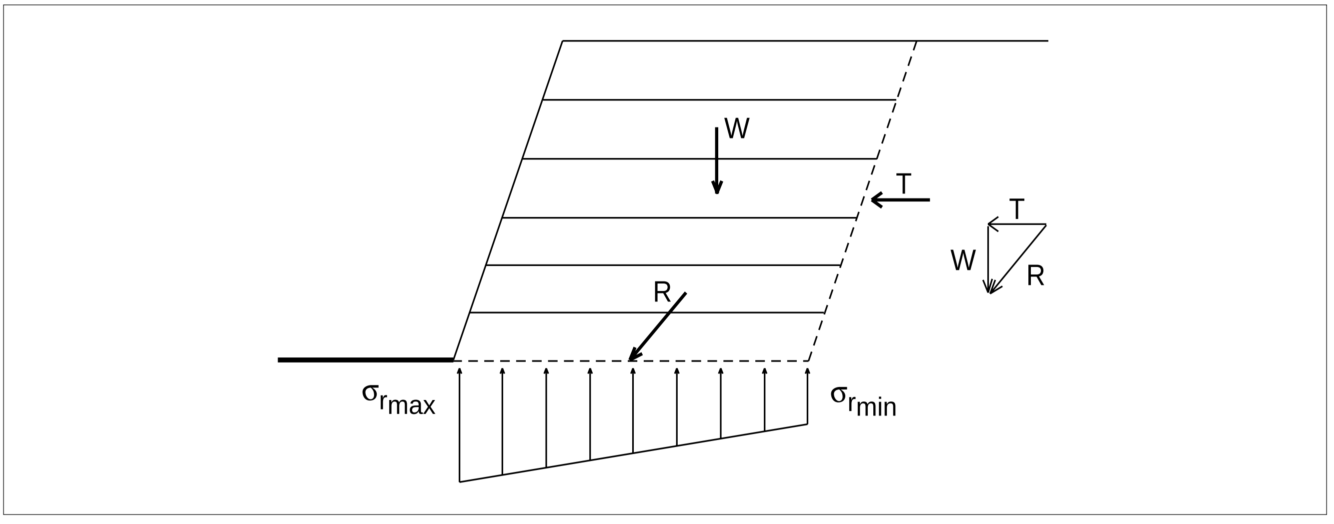


Fig. 12 - Equilibrium of the reinforcement zone, acting as a rigid block

5) The distribution of the reinforcement layers must satisfy the equilibrium for every possible failure mechanism still remaining under the design strength P. The possible failure mechanisms might be planar, multi-wedge, circular or logarithmic.

Extensive investigations suggest that the force equilibrium of a two-parts wedge mechanism may be used to determine the gross maximum required force for equilibrium with sufficient accuracy (see fig.13). Then we need to find the critical two-parts wedge mechanism that gives the maximum required force and to design the reinforcement on the basis of the maximum required force and of the design strength P.

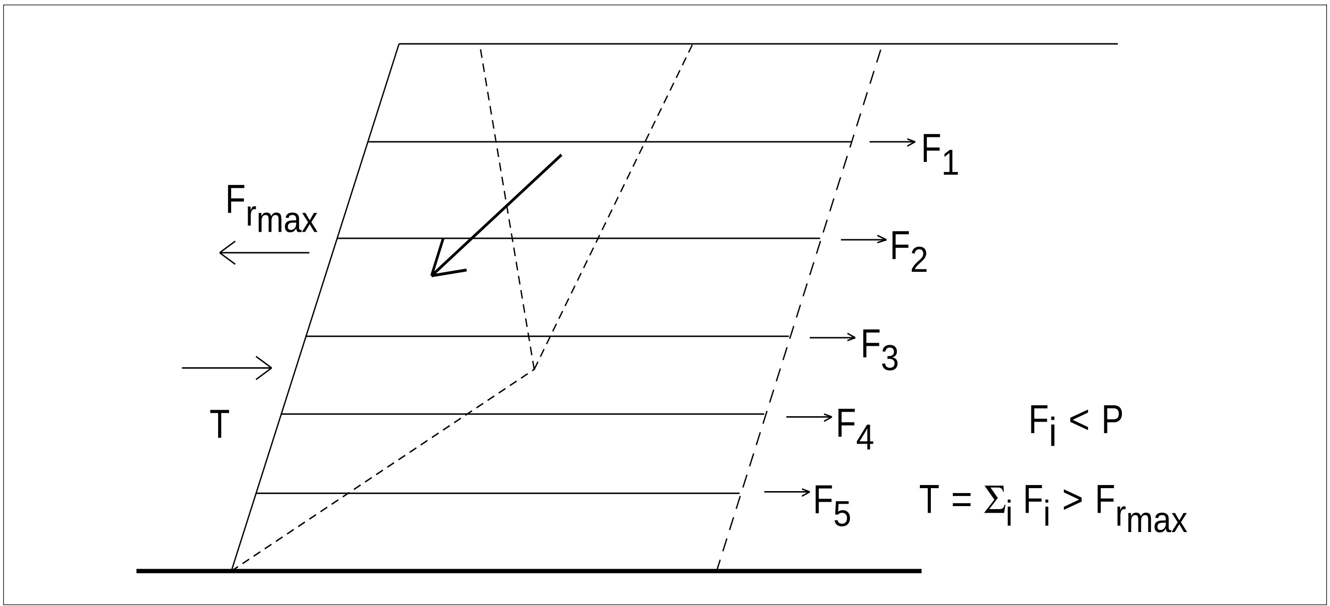


Fig. 13 - Two-parts wedge mechanism

8 Gross required force

The determination of the gross force required for equilibrium is essentially an earth pressure problem and may be conveniently approached by using the Limit Equilibrium Method based on the two-parts wedge mechanism.

Referring to fig. 14, the steps in this procedure are:

- set the node of the two blocks so the angle θ_1 is set;
- sistematically varying the angle θ_2 and imposing the equilibrium of forces, calculate the required force T, so that the maximum required force is obtained for every possible mechanism with the previously set node;
- repeat the calculation for a whole grid of nodes until the gross maximum required force T_{max} is found.

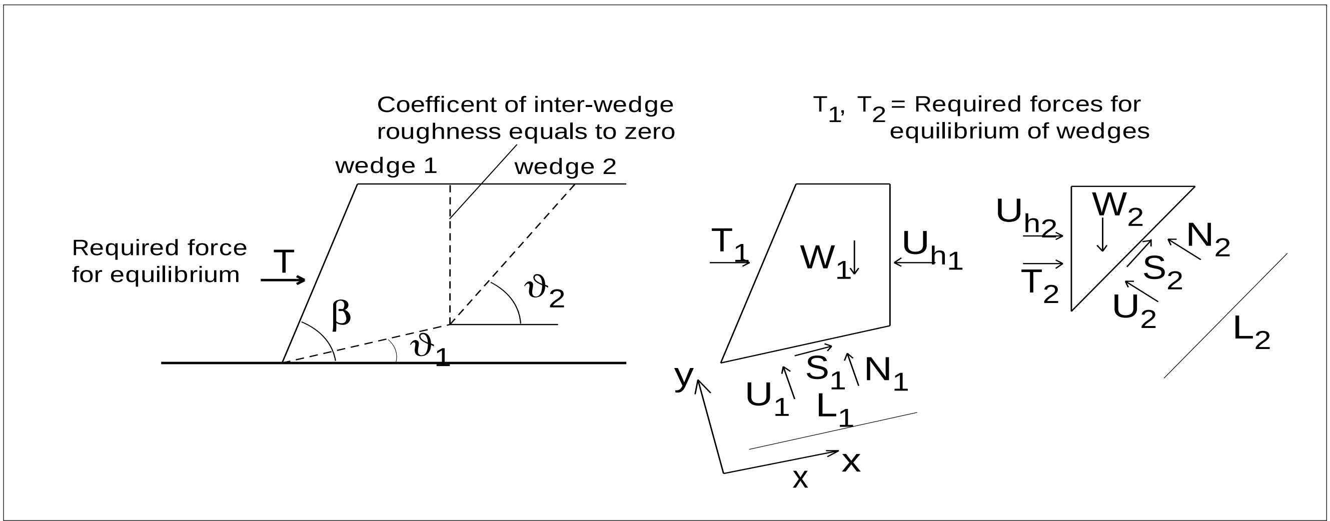


Fig. 14 - Calculation of the gross maximum required force by a two-parts wedge mechanism

Always referring to fig. 14, we have:

$$U_{h1} = -U_{h2} \tag{21}$$

$$T_1 = \frac{W_1 \cdot (\tan\theta_1 - \tan\phi') - \left(c' \cdot \frac{l_1}{\cos\theta_1}\right) + U_1 \cdot \frac{\tan\phi'}{\cos\theta_1}}{1 + \tan\theta_1 \cdot \tan\phi'} + U_{h1} \tag{22}$$

$$T_2 = \frac{W_2 \cdot (\tan\theta_2 - \tan\phi') - \left(c' \cdot \frac{l_2}{\cos\theta_2}\right) + U_2 \cdot \frac{\tan\phi'}{\cos\theta_2}}{1 + \tan\theta_2 \cdot \tan\phi'} + U_{h2} \tag{23}$$

$$T = T_1 + T_2 \tag{24}$$

with: c', f' = effective cohesion and friction angle of soil.
It is to be noticed that, thanks to eq. (22), (23), (24) we can write:

$$T = W \cdot f_1 \cdot (\phi', \theta_1, \theta_2) - c' \cdot f_2 \cdot (\phi', \theta_1, \theta_2, l_1, l_2) + u \cdot f_3 \cdot (\phi', \theta_1, \theta_2) \tag{25}$$

with f_1, f_2, f_3 = functions of the parameters in brackets.

T can be expressed in terms of an earth pressure coefficient K by using the Rankine-like active pressure equation:

$$T = 1/2 \cdot K \cdot \gamma \cdot H^2 \quad (26)$$

with:

K = coefficient of earth pressure in terms of β, ϕ
γ = unit weight of soil
H = height of the slope

We could also write:

$$\frac{T}{\gamma} = \frac{W}{\gamma} \cdot f_1 \cdot (\phi', \theta_1, \theta_2) - \frac{c'}{\gamma} \cdot f_2 \cdot (\phi', \theta_1, \theta_2, l_1, l_2) + \frac{u}{\gamma} \cdot f_3 \cdot (\phi', \theta_1, \theta_2) \quad (27)$$

But:

$$W / \gamma = V = A \cdot 1 \text{ meter}$$

being: V = volume of the two wedges
A = area of the vertical section of the two wedges

if we set:

$$r_u = \frac{u}{\gamma \cdot H} = \text{pore water coefficient}$$

$$c' = 0$$

eq. (27) becomes:

$$\frac{T}{\gamma} = A \cdot f_1 \cdot (\phi', \theta_1, \theta_2) + r_u \cdot f_3' \cdot (\phi', \theta_1, \theta_2) \quad (28)$$

with: f_3' function of the parameter in brackets.
On the basis of (26) and (28), it results:

$$K = \frac{2}{H^2} \cdot \frac{T}{\gamma} \quad (29)$$

If we then set H = 1, we finally have:

$$K = 2 \cdot \frac{T}{\gamma} \qquad (30)$$

Therefore it is possible to obtain the soil pressure coefficient, K, by applying the two-parts-wedge limit equilibrium method to a slope of unitary height and calculating (T/γ) with eq.(28) and K with eq. (30).
By means of a sistematic calculation with a computer program, it is possible to obtain the diagram of K versus the slope angle β and the angle of internal friction of the soil ϕ'. Usually one graph for each value of r_u is obtained. It is also possible to obtain these diagrams taking into account a cohesion of the soil $c' > 0$; this work has not yet been done, but anyway it is in favour of safety to assume $c' = 0$.

9 Design charts

The results of the calculations done following the described criteria can be plotted as design charts. The design charts allow to determine the earth pressure coefficient K and the length of reinforcement, L, as a function of the slope angle β, of the soil friction angle ϕ' and of the pore water pressure parameter r_u. New design charts have been recently presented (Jewell, 1991); they are shown in fig. 15, 16, 17.
These charts are applicable to steep slopes reinforced with geogrids so that:

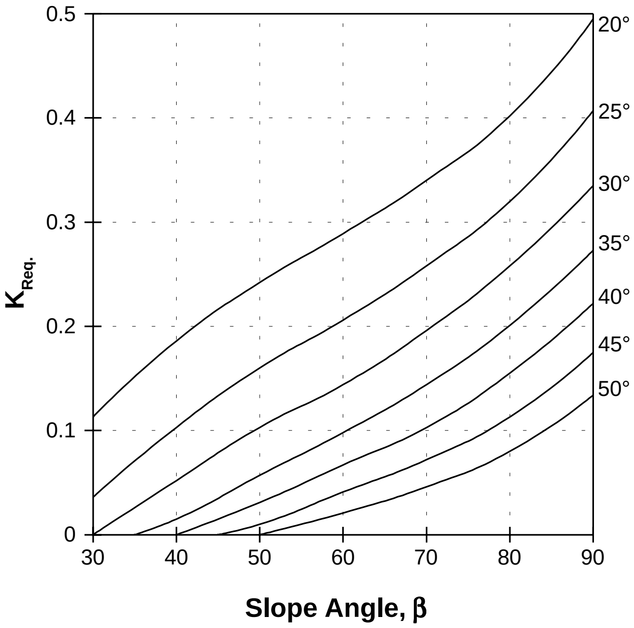
- the slope is uniform with a horizontal crest and a slope angle in the range of 30° to 90°;
- the foundation is levelled and with adequate bearing capacity;
- the fill material is of a single type;
- the fill characteristics are expressed in terms of effective stresses, with zero cohesion ($c'=0$);
- pore water pressures (if present) are expressed in terms of the coefficient $r_u = u/(z \cdot \gamma)$;
- surcharge loading on the crest (if present) is uniformly distributed;
- the reinforcement is continuous (eg sheet or grid) and is placed horizontally in the fill.

The charts do not allow for:

- totally submerged slopes;
- point or line loading on the crest, or loading on the slope face;
- dynamic loading;
- soil shear strength expressed in terms of total stresses ($c_u \geq 0$ and $\phi_u = 0$);
- discrete reinforcing inclusions such as strips or bars.

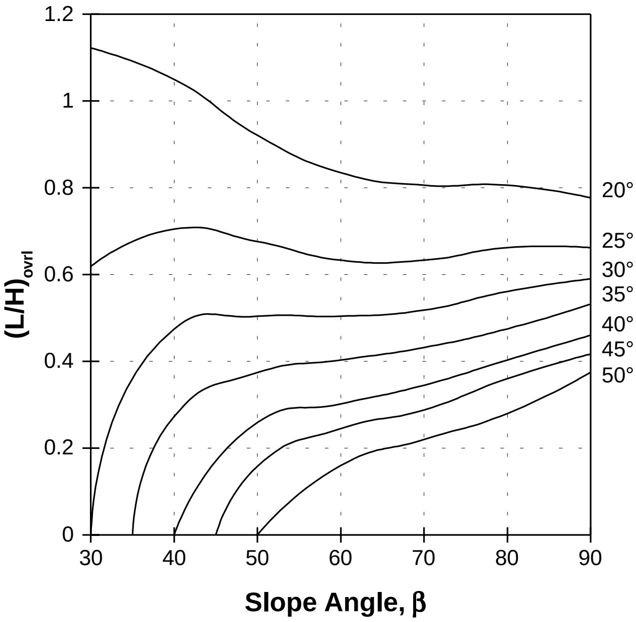
Steep Reinforced Slope Design Charts (Jewell, 1991)

Minimum Required Force $K_{Req.}$



R_u=u/(γ z)=0.00

Minimum Required Length
Overall Stability



Minimum Required Length
Direct Sliding

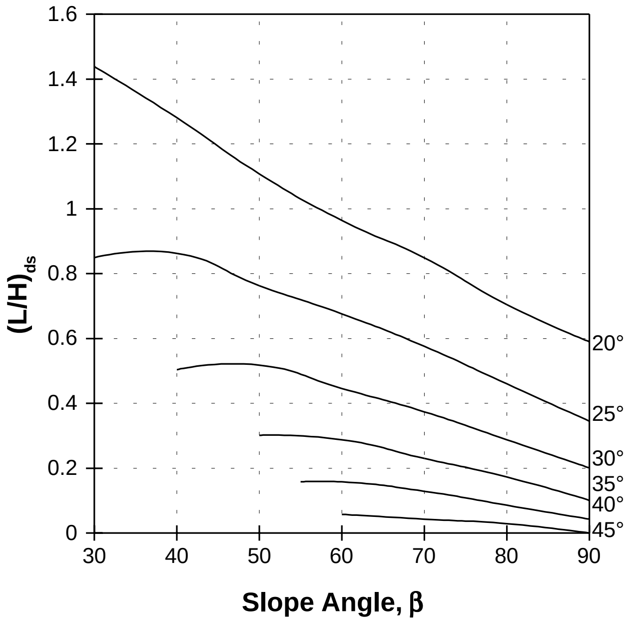
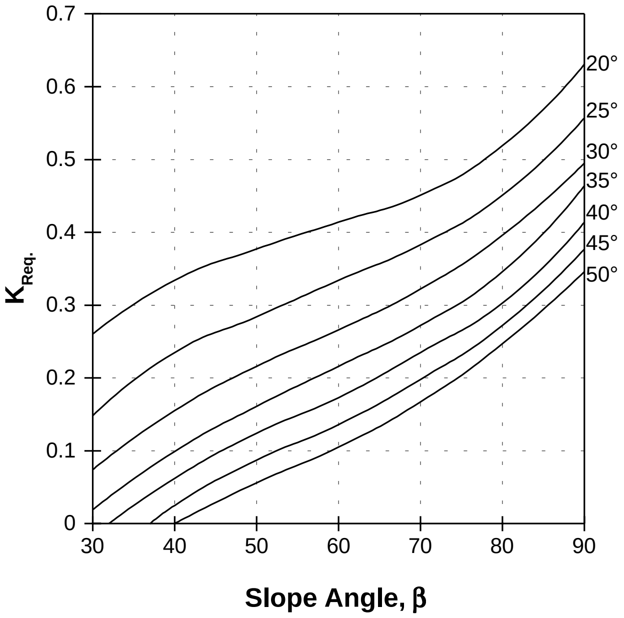


Fig. 15

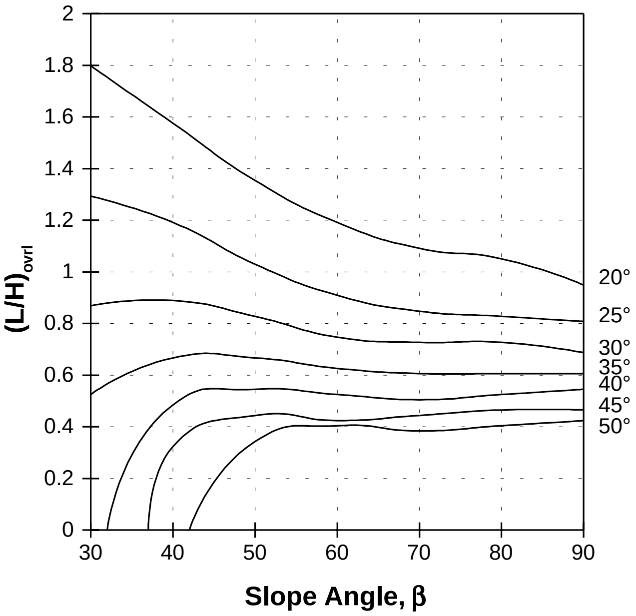
Steep Reinforced Slope Design Charts (Jewell, 1991)

Minimum Required Force $K_{Req.}$



R_u=u/(γ z)=0.25

Minimum Required Length
Overall Stability



Minimum Required Length
Direct Sliding

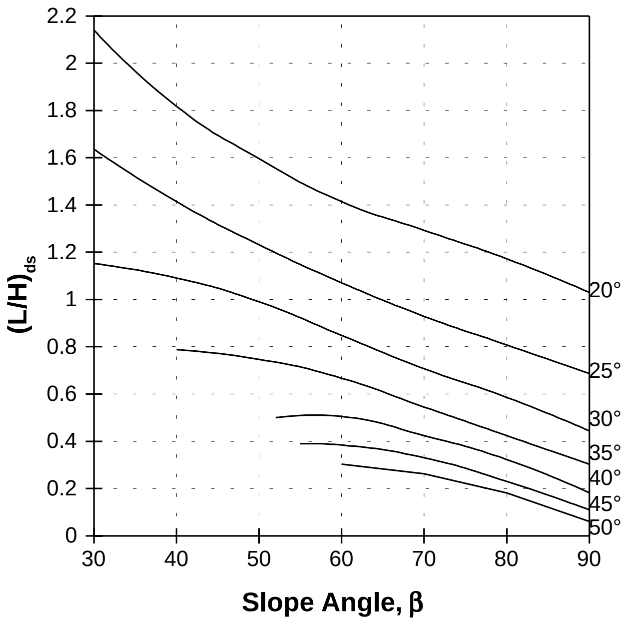
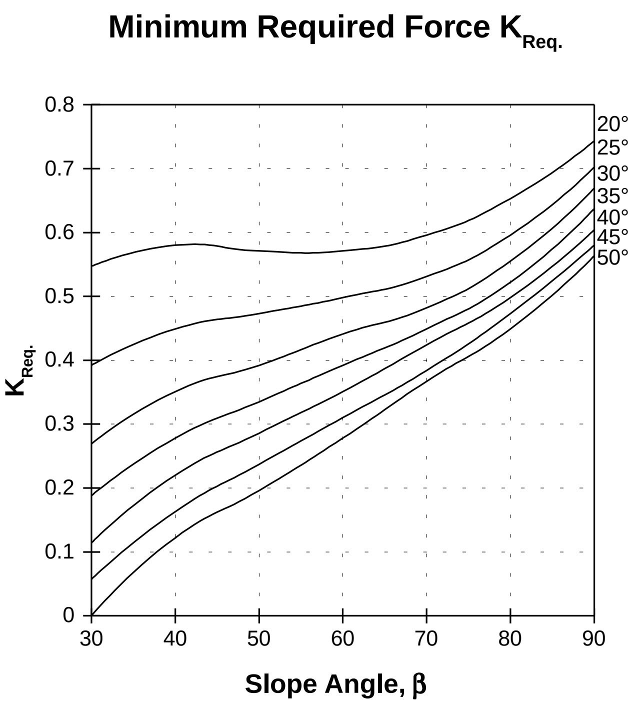
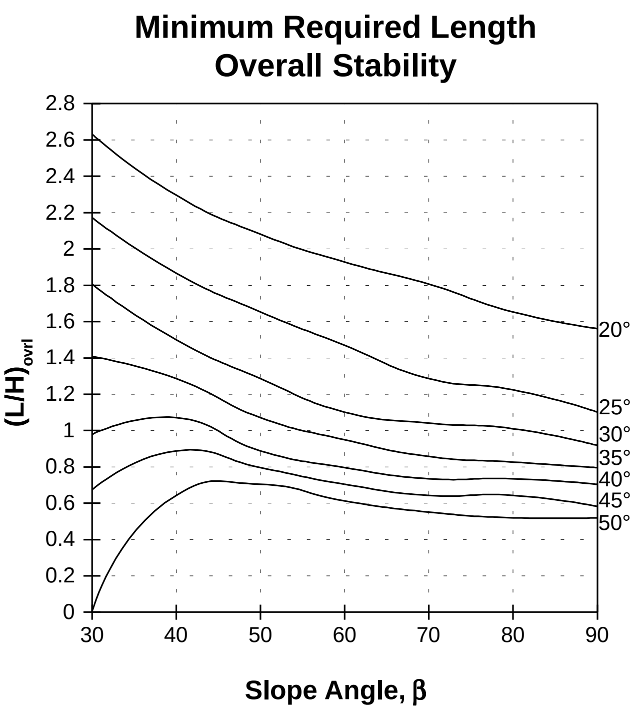


Fig. 16

Steep Reinforced Slope Design Charts (Jewell, 1991)



R_u=u/(γ z)=0.50



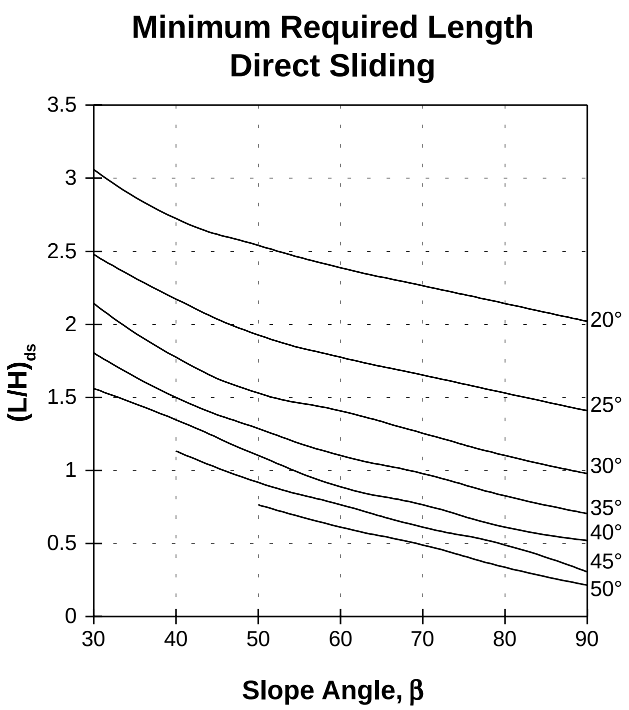


Fig. 17

10 Procedure to determine reinforcement spacing

As we already said, it would be prudent to space reinforcement layers on the assumption that each layer locally have to support the same horizontal force. The maximum expected local force in the reinforcement has the value:

$$F_r = S_v \cdot K \cdot \gamma \cdot z = n \cdot v \cdot K \cdot \gamma \cdot Z \qquad (31)$$

with: n = number of compaction lifts for every spacing
S_V = vertical spacing

The calculation of the spacing arrangements for the reinforcement is simplified by defining a spacing costant Q for the slope in terms of the minimum spacing v to be used:

$$Q = \frac{P}{K \cdot \gamma \cdot v} \qquad (32)$$

and then:

$$P = v \cdot K \cdot \gamma \cdot Q \qquad (33)$$

from (31) and (33):

$$F_r = P \cdot \frac{S_v}{v} \cdot \frac{z}{Q} \qquad (34)$$

Q results equal to the depth to which the required vertical spacing is just v.
The allowable strength should be greater than the required force F_r:

$$P \geq F_r \qquad (35)$$

that is:

$$\frac{v}{S_v} \geq \frac{z}{Q} \qquad (36)$$

Equation (36) gives the ideal spacing that optimizes the geogrid strength.

For example,	if Z/Q = 0.5	then	v/Sv	=	0.5;
	if Z = Q/2	then	Sv	=	2v
	if Z = Q/3	then	Sv	=	3v

Hence it is possible to define zones where the reinforcement layers are equally spaced; these zones shall respect the conditions expressed by eq. (35) and eq. (36).
It is useful to choose the depths of every spacing zone as fractions of the constant Q:

$$z_0 = Q; \quad z_1 = Q/2; \quad z_2 = Q/3; . \quad . \tag{37}$$

11 Wrapping technique

To prevent local slumping and face erosion, it is possible to build the face of the embankment with precast concrete panels, concrete blocks or wrapping the grids around the face.
Using the wrap-around technique the wrapping length L_r of the geogrids must be calculated.
The problem is solved by imposing that the local thrust of the soil, applied on every face section of the geogrids, does not pullout the horizontal wrapped length.
Referring to fig. 18 the equilibrium is imposed increasing the outward thrust with the factor of safety FS_{wrap}:

$$FS_{wrap} \cdot K \cdot \gamma \cdot (z_i + S_{v_j} / 2) \cdot S_{v_j} = \gamma \cdot z_i \cdot f_{ds} \cdot \tan\phi ' \cdot L_{r_i} \tag{38}$$

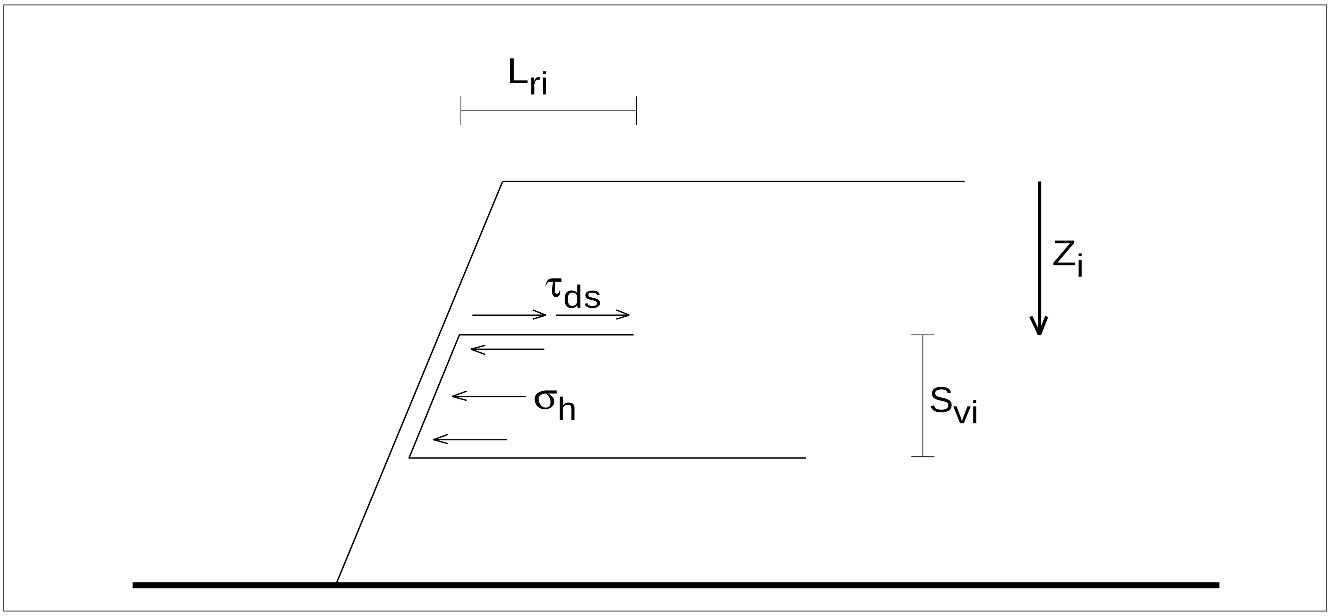


Fig. 18 - Scheme for calculation of the wrapping length

In Fig. 18, σ_h and τ_{ds} are given by:

$$\sigma_h = K \cdot \gamma \cdot (z_i + S_{v_j} / 2)$$
$$\tau_{ds} = \gamma \cdot z_i \cdot f_{ds} \cdot \tan\phi '$$

hence:

$$L_{r_i} = \frac{FS_{wrap} \cdot K \cdot (z_i + S_{v_j} / 2) \cdot S_{v_j}}{z_i \cdot f_{ds} \cdot \tan\phi '} \qquad (39)$$

The Factor of Safety FS_{wrap} can usually be assumed to be in the range 1.20 ÷ 1.40.

12 Design chart procedure

The steps for the design of a reinforced slope, referring to the charts in fig. 15, 16, 17 are:

1) Define the geometrical configuration of the slope and eventually the uniformly distributed surcharge loading on the top of the slope W_S, as in fig. 19.

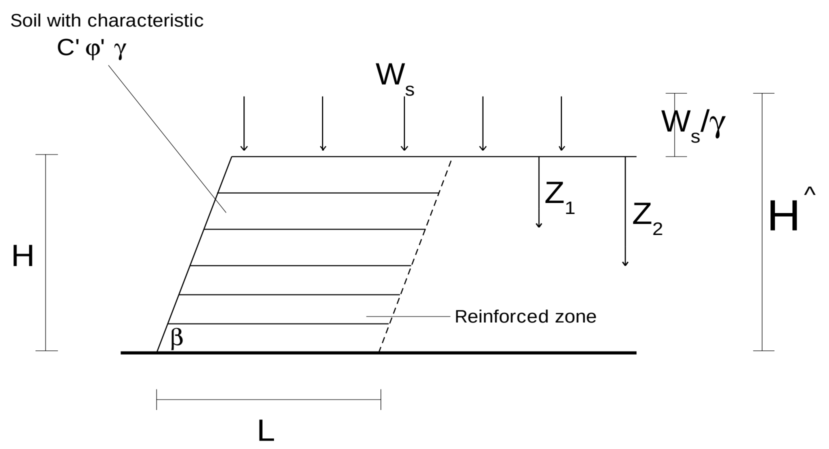


Fig. 19 - Steep slope: definition of symbols

Calculate the apparent height (H^), where:

$$\hat{H} = H + (W_s / \gamma) \qquad (40)$$

2) Set the Safety Factors FS_{grid} (see tab. 1, 2, 3, 4, 5).

Calculate the allowable resistance P for the reinforcement using eq. (17), (18) in terms of the LTDS of the grid.

3) Define the parameters ϕ' and γ of the soil and the maximum pore water pressure r_u:

$$r_u = \max[u(z) / z \cdot \gamma] \quad (41)$$

where:

u(z) = pore water pressure at the depth z under the crest of the slope.

4) Using the value of the slope angle β and the angle of friction ϕ' of the soil, calculate the coefficient of earth pressure K and the ratios of reinforcement length to embankment height $(L/H)_{ovrl}$ and $(L/H)_{ds}$ using one of the set of charts in fig. 15, 16, 17. The chart to be used shall be selected on the base of the anticipated value of the pore water pressure coefficient r_u. Select the required reinforcement length as follows:

a) if $(L/H)_{ovrl} > (L/H)_{ds}$

the reinforcement length shall be constant and equal to:

$$L = \hat{H} \cdot (L / H)_{ovrl} \quad (42)$$

b) if $(L/H)_{ovrl} < (L/H)_{ds}$

the reinforcement length can be:

b1) constant and equal to:

$$L = \hat{H} \cdot (L / H)_{ds} \quad (43)$$

b2) with a length varying uniformly from:
- length at the base:

$$L = \hat{H} \cdot (L / H)_{ds} \quad (44)$$

to:
- length at the crest:

$$L = \hat{H} \cdot (L / H)_{ovrl} \quad (45)$$

5) Select the minimum vertical spacing v for a single layer of compacted soil and calculate the spacing costant Q referring to eq. (32).

6) Define the zones for reinforcement layers spaced equally at v, 2v, 3v... as shown in tab. 6.

Tab. 6 - Reinforcement spacing

SPACING S_{vi} [m]	**DEPTH Z_i [m]**	**THICKNESS s_i [m]**
S_{v1} = v	Q ÷ Q/2	s_1 = H^ - Q/2
S_{v2} = 2v	Q/2 ÷ Q/3	s_2 = Q/2 - Q/3
. . .	. . .	. . .
S_{vn} = nv	Q/n ÷ Ws/	s_n = Q/n - Ws/

Important note:

If H^<Q the minimum spacing v at the base of the slope will have to be reduced or a more resistant geogrid has to be selected.

7) Calculate the number of required reinforcement layers: the first layer is placed on the foundation at the base of the slope, the other required layers are calculated starting from the base. Referring to tab. 7, the steps of the procedure are:

• divide the thickness of every zone (see tab. 7) by the spacing of the reinforcement layers in that zone to calculate the number of grids in a zone. The result is rounded down to the nearest whole number.

$$N_i = (\widehat{s}_i \ / \ S_v)_{whole\ number} \tag{46}$$

• calculate the remaining thickness of the zone:

$$R_i = \widehat{s}_i - S_v \cdot N_i \tag{47}$$

• add R_i to the thickness of the next zone:

$$\hat{s}_{i+1} = s_{i+1} + R_i \qquad (48)$$

with:

$$R_0 = 0$$

- Repeat the calculation for all the zones.

Tab. 7 - Calculation of the required layers

S_i ^/Sv_i	N_i	R_i [m]	s_{i+1} ^ [m]
S_1 ^/SV_1	N_1 = INT (s_1 ^/Sv_1)	$R_1 = s_1$^ - Sv_1 N_1	s_1 ^ = $s_1 + R_1$
S_2^/Sv_i	N_2 = INT (s_2^/Sv_2)	$R_2 = s_2$^ - $Sv_2 \cdot N_2$	s_2^ = $s_2 + R_2$
S_3 ^/Sv_3	N_3 = INT (s_3 ^/Sv_3)	$R_3 = s_3$^ - $Sv_3 \cdot N_3$	s_3^ = $s_3 + R_3$
. . . .	. . . .	. . . .	. . . .
S_n ^/Sv_n	N_n = INT (s_n^/S_n)	$R_n = s_n$^ - Sv_n N_n	
N_{tot} = 1 + $N_1 + N_2 + N_3$ + ... + N_n			

Note

- If the top layer of reinforcement is more than 0.6 m below the slope crest it would be prudent to add an addictional layer near to the crest.

8) Calculate the gross horizontal force required for equilibrium:

$$T = 1/2 \cdot K \cdot \gamma \cdot \widehat{H}^2 \quad (49)$$

9) Verify that the average required force for every layer is less than the safe design strength of the grid:

$$(T / N_{tot}) \leq P \quad (50)$$

where N_{tot} = total number of geogrids.
If this condition is not verified, increase the number of geogrid layers or repeat the procedure changing the minimum spacing.

10) When using the wrap-around technique, calculate the wrapping length L_r for every layer:

$$\widehat{z}_i = z_i + (W_s / \gamma) \quad (51)$$

$$L_{r_i} = \frac{FS_{wrap} \cdot K \cdot (\widehat{z}_i + S_{v_j} / 2) \cdot S_{v_j}}{\widehat{z}_i \cdot f_{ds} \cdot \tan \widehat{\phi}\,'} \quad (52)$$

$$L_r = \max[L_{r_i}] \qquad \text{optionally} \quad (53)$$

11) Draw the final layout of the reinforced slope.

13 Worked example

We have to reinforce the slopes of a steep sided embankment: it is 6.0 m high, very wide with a slope angle of 70°. The maximum surcharge load on the crest is 10 kN/m². The foundation is levelled and with adequate bearing capacity. The design life is 50 years. The soil for the construction is a sandy gravel with a small percentage of silt, available in situ, having the following characteristics:

$$c' = 10 \ kPa \qquad \phi\,' = 32° \qquad \gamma = 20 \ kN/m^3$$

We have to consider that the embankment may be wet for long periods because the rainfall in the region is quite high.
We want to reinforce the steep slope of the embankment with, for example, TENAX TT301 geogrids.

The data of the problem are summarized in fig. 20.

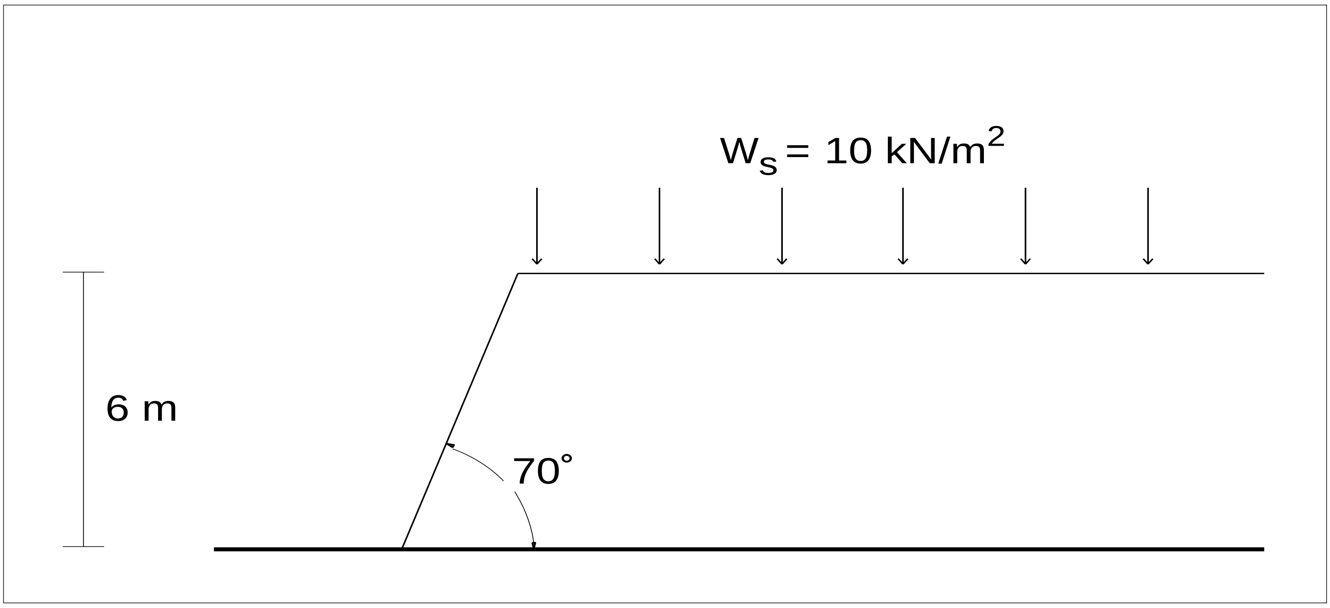


Fig. 20 - Design data of the example and section of the embankment to be reinforced

Following the procedure explained at paragraph 12,, the design is developed as following:

1) We set the input values:

$H = 6.0 \ m$

$\beta = 70°$

$W_S = 10 \ kN/m^2$

$\gamma = 20 \ kN/m^3$

$\varnothing = 34°$

hence:

$\hat{H} = H + (W_s / \gamma) = 6.5 \ m$

2) The following values are selected:

$FS_{overtension}$	=	1.30 (project of average difficulty and dimension)
$FS_{construction}$	=	1.00 (sandy gravel soil)
$FS_{biological}$	=	1.00
$FS_{chemical}$	=	1.00
$FS_{junction}$	=	1.00

Then we have:

$$FS_{grid} = FS_{creep} \cdot FS_{junction} \cdot FS_{construction} \cdot FS_{chemical} \cdot FS_{biological} = 2.76 \cdot 1.00 \cdot 1.00 \cdot 1.00 \cdot 1.00 = 2.76$$

The allowable strength of TENAX TT 301 geogrids is:

$$T_{allow} = \frac{T_{ult}}{FS_{grid}} = 23.5\ kN/m$$

The design strength results:

$$P = T_{allow} / FS_{grid} = 23.5 / 1.30 = 18.1\ kN/m$$

3) We assume:

$$c' = 0\ kPa \qquad \phi' = 34°$$

In this case we do not know u(z). In order to take into account the pore water pressure which may develop during periods of intense rainfall, it is better to assume:

$$r_u = 0.25$$

4) From fig. 22, related to r_u = 0.25, we obtain:

$$K = 0.28$$

$$\left(\frac{L}{H}\right)_{ovrl} = 0.63$$

$$\left(\frac{L}{H}\right)_{ds} = 0.58$$

therefore:

$$L = \hat{H} \cdot \left(\frac{L}{H}\right)_{ovrl} = 4.10$$

5) We choose a compaction lift of 0.30 m; accordingly it will be:

$$v = 0.30\ m$$

then:

$$Q = \frac{P}{K \cdot \gamma \cdot v} = \frac{18.1}{0.28 \cdot 20 \cdot 0.30} = 10.68$$

6) Calculate the zones of equal spacing:

Sv_i [m]	Z_i [m]	s_i [m]
$Sv_1 = 0.3$	$Q \div Q/2 = 10.68 \div 5.34$	$S_1 = 6.50 - 5.34 = 1.16$
$Sv_2 = 0.6$	$Q/2 \div Q/3 = 5.34 \div 3.56$	$S_2 = 5.34 - 3.53 = 1.78$
$Sv_3 = 0.9$	$Q/3 \div Ws/\gamma = 3.56 \div 0.50$	$S_3 = 3.56 - 0.50 = 3.06$

7) Calculate the number and position of the required layers:

$s_i\hat{}/sv_i$	N_i	R_i [m]	$s_{i+1}\hat{}$ [m]
		$R_0 = 0.0$	$s_1\hat{} = 1.16$
$1.16/0.30 = 3.86$	$N_1 = 3$	$R_1 = 1.16 - 3 \cdot 0.30 = 0.26$	$s_2\hat{} = 2.04$
$2.04/0.60 = 3.40$	$N_2 = 3$	$R_2 = 2.04 - 3 \cdot 0.60 = 0.24$	$s_3\hat{} = 3.30$
$3.30/0.90 = 3.67$	$N_3 = 3$	$R_3 = 3.30 - 3 \cdot 0.90 = 0.60$	
$N_{tot} = 3 + 3 + 3 = 9$			

We add a 10th layer spaced 0.60 m near to the crest.

8) Calculate the gross horizontal force for equilibrium:

$$T = \frac{1}{2} \cdot K \cdot \gamma \cdot \hat{H}^2 = 119.15\ kN / m$$

9) Check the average tensile force in the geogrids:

$$\frac{T}{N_{tot}} = \frac{119.15}{10} = 11.91\ kN / m$$

$$P = 18.08\ kN / m$$

$$T \ / \ N_{tot} \ \leq \ P$$

10) Calculate the wrapping length.
Conservatively: we set for this type of soil:
For the lower layer:

$$\hat{z}_1 = z_1 + \left(\frac{W_s}{\gamma}\right) = 5.7 + 0.5 = 6.2\ m$$

$$L_{r_1} = \frac{FS_{wrap} \cdot K \cdot \left(\hat{z}_1 + \frac{S_{v_1}}{2}\right) \cdot S_{v_1}}{\hat{z}_1 \cdot f_{ds} \cdot \tan\phi'} = \frac{1.30 \cdot 0.28 \cdot \left(6.2 + \frac{0.30}{2}\right) \cdot 0.30}{6.2 \cdot 0.85 \cdot \tan 34°} = 0.21\ m$$

... for the 10th layer:

$$\hat{z}_{10} = z_{10} + \left(\frac{W_s}{\gamma}\right) = 0.6 + 0.5 = 1.1\ m$$

$$L_{r_{10}} = \frac{FS_{wrap} \cdot K \cdot \left(\hat{z}_{10} + \frac{S_{v_{10}}}{2}\right) \cdot S_{v_{10}}}{\hat{z}_{10} \cdot f_{ds} \cdot \tan\phi'} = \frac{1.30 \cdot 0.28 \cdot \left(1.1 + \frac{0.90}{2}\right) \cdot 0.90}{0.60 \cdot 0.85 \cdot \tan 34°} = 0.87\ m$$

We impose that the minumum wrapping length is 1.5 meter, so it is necessary to consider a wrapping length of 1.5 for each layer.

11) The final layout is shown in Fig. 21.

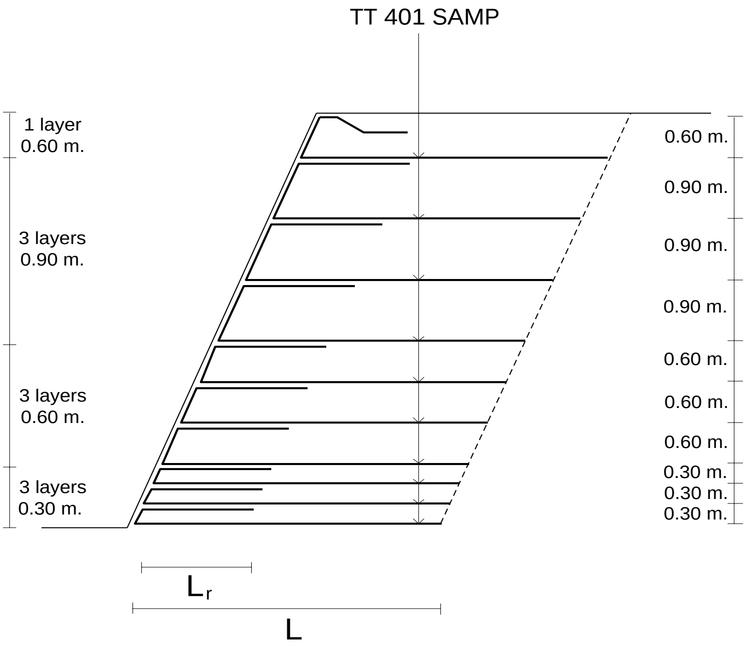


Fig. 21 - Final layout for the worked example

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